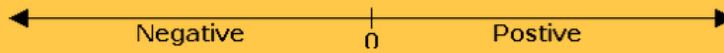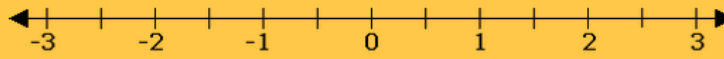# Number Lines

A **number line** is a line representing the set of all real numbers.

Number lines are typically marked showing integer values
with smaller numbers on the left and larger numbers on the right.

On a number line, all numbers to the right of zero are positive,
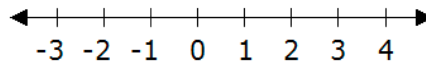and all the numbers to the left of zero are negative.

Fraction and **decimal** values can also be marked and/or determined on a number line.

Opposite Numbers

Example 1:

What is the opposite of 2?

Solution:

The opposite of a number is the number that is the same distance from 0 on the number line, but in the opposite direction.

Since 2 is 2 units to the right of 0, the only number that is the same distance from 0 is the number that is 2 units to the left of 0.

So, the opposite of 2 is **-2**.

Integers

Example 2:

What value does the red dot represent on the number line?
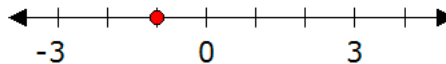
Solution:

On this number line, each tick mark is a whole number. The red dot is 1 tick mark to the left of 0.

Moving to the right, the numbers increase. Moving to the left, the numbers decrease.

So, the red dot represents **-1**.

Fractions
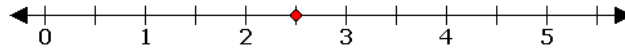
Example 3:

What value does the red dot represent on the number line?

A number line labeled 0, 1, 2, 3, 4, 5 with a red dot halfway between 2 and 3.

Solution:

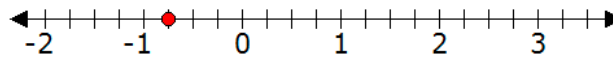The numbers given on the number line are consecutive whole numbers.

The red dot is between 2 and 3. There are two equal parts between each number, so each tick mark represents $\frac{1}{2}$.

The red dot is halfway between 2 and 3, so the red dot represents $2\frac{1}{2}$.

Decimals

Example 4:

What value does the red dot represent on the number line?

A number line labeled -2, -1, 0, 1, 2, 3 with a red dot between -1 and 0.

Solution:

The numbers given on the number line are consecutive integers.

The red dot is between -1 and 0. There are four equal parts between each number, so each tick mark represents one-fourth (or 0.25).

The red dot is on the third tick mark to the left of 0, so the red dot represents negative three-fourths, or -0.75.

VIDEO: https://www.khanacademy.org/math/arithmetic/decimals/decimals-on-number-line/v/points-on-a-number-line?v=uC09taczvOo

EXAMPLES:

1.

A number line labeled 0 to 1 with a red dot.
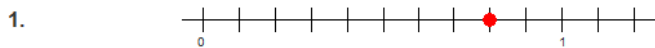
What value does the red dot represent on the number line?

- A) $\frac{4}{5}$
- B) 4
- C) $1\frac{3}{5}$
- D) 8

2. What is the opposite of -0.1?

- A) 0.2
- B) 0
- C) -0.2
- D) 0.1

Arithmetic with Whole Numbers

Addition

- *Place the numbers so that the ones place of each number is aligned.*

- *Add each column, starting on the right and working left.*

- *If the sum of a column is greater than ten, then carry the one to the next column on the left.*

Subtraction

- *Place the numbers so that the ones place of each number is aligned.*

- *Subtract each column, starting on the right and working left.*

- *When subtracting two numbers in a column, borrowing is needed if the number in the top row is smaller than the number in the bottom row.*

- *For example, when borrowing is needed for the ones column, a ten in the tens column can be exchanged for 10 ones to be added to the ones column.*

Example 1:

$$853 - 227 = ?$$

Solution:

$$
\begin{array}{r}
{}^{4}\,{}^{13} \\
8\cancel{5}\cancel{3} \\
-\ 227 \\
\hline
626
\end{array}
$$

Arithmetic with Whole Numbers

To multiply a four-digit number by a two-digit number, follow the steps below.

Example:

Solve the following.

Solution:

$$
\begin{array}{r}
2{,}415 \\
\times\ 25 \\
\hline
\end{array}
$$

Multiply the two numbers in the ones places: 5 × 5 = 25. Since this number is 10 or greater, place the two above the tens place column and place the five below the line in the ones place column.

$$
\begin{array}{r}
^{2} \\
2{,}41\mathbf{5} \\
\times\ 2\mathbf{5} \\
\hline
5 \\
\end{array}
$$

Multiply the digit in the tens place column of the top number, 1, by the digit in the ones place column of the bottom number ,5: 1 × 5 = 5. Now, add the 2 that was carried over from the previous multiplication. Since this number is less than 10, place the five below the line in the tens place column.

$$
\begin{array}{r}
^{2} \\
2{,}4\mathbf{15} \\
\times\ 25 \\
\hline
75 \\
\end{array}
$$

Multiply the digit in the hundreds place column of the top number, 4 by the digit in the ones place column of the bottom number, 5: 4 × 5 = 20. Since this number is 10 or greater, place the two above the thousands place column and place the zero below the line in the hundreds place column.

$$
\begin{array}{r}
^{2} \\
2{,}\mathbf{4}15 \\
\times\ 25 \\
\hline
\mathbf{0}75 \\
\end{array}
$$

Multiply the digit in the thousands place column of the top number, 2 by the digit in the ones place column of the bottom number, 5: 2 × 5 = 10. Now, add the 2 that was carried over from the previous multiplication. Since this is the last number on the top, place the 12 to the left of the digits under the line.

$$
\begin{array}{r}
^{2} \\
\mathbf{2}{,}415 \\
\times\ 25 \\
\hline
\mathbf{12}{,}075 \\
\end{array}
$$

After 2,415 has been multiplied by 5 as shown above, multiply 2,415 tens place of the second number, 2. The resulting number is moved one place to the left because 2,415 is being multiplied by a tens place number. Place a zero in the ones column to indicate this.

$$
\begin{array}{r}
2{,}415 \\
\times\ 25 \\
\hline
12{,}075 \\
\mathbf{0} \\
\end{array}
$$

Multiply the digit in the ones place column of the top number, 5, by the digit in the tens place column of the bottom number ,2: 5 × 2 = 10. Since this number is 10 or greater, place the zero below the line in the tens place column and place the one above the one in the tens place column of the top number.

$$
\begin{array}{r}
^{1} \\
2{,}4\mathbf{15} \\
\times\ \mathbf{2}5 \\
\hline
12{,}075 \\
\mathbf{00} \\
\end{array}
$$

Multiply the digit in the tens place column of the top number, 1, by the digit in the tens place column of the bottom number, 2: 1 × 2 = 2. Now, add the 1 that was carried over from the previous multiplication. Since this number is less than 10, place the three below the line in the hundreds place column.

$$
\begin{array}{r}
{\color{red}1} \\
2{,}41{\color{red}5} \\
\times\ 25 \\
\hline
12{,}075 \\
{\bf 3}00
\end{array}
$$

Multiply the digit in the hundreds place column of the top number, 4, by the digit in the tens place column of the bottom number, 2: 4 × 2 = 8. Since this number is less than 10, place the eight below the line in the thousands place column.

$$
\begin{array}{r}
2{,}415 \\
\times\ 25 \\
\hline
12{,}075 \\
{\bf 8}{,}300
\end{array}
$$

Multiply the digit in the thousands place column of the top number, 2, by the digit in the tens place column of the bottom number, 2: 2 × 2 = 4. Since this number is less than 10, place the four below the line in the ten-thousands place column.

$$
\begin{array}{r}
2{,}415 \\
\times\ 25 \\
\hline
12{,}075 \\
{\bf 4}8{,}300
\end{array}
$$

Finally, add 12,075 and 48,300.

$$
\begin{array}{r}
2{,}415 \\
\times\ 25 \\
\hline
12{,}075 \\
+\ 48{,}300 \\
\hline
{\bf 60{,}375}
\end{array}
$$

Division of Whole Numbers

DIVISION is the opposite operation of multiplication, so that the following is true.

If a × b = c, then a = c ÷ b.

In the equation a = c ÷ b, a is the **quotient**, c is the **dividend**, and b is the **divisor**.

quotient = dividend ÷ divisor

When dividing, the order of the numbers does matter.

35 ÷ 7 does not equal 7 ÷ 35

Division has special properties with 1 and 0.

Any number divided by one is equal to the number.

4,567 ÷ 1 = 4,567

Division by zero is not defined.

4,567 ÷ 0 = undefined

Example - Divide 2,067 by 3

- Dividing a four digit number by a one digit number requires several steps. Place the divisor (3) before the division bracket and place the dividend (2,067) in the division bracket.

$$3\overline{)2067}$$

- The first number of the dividend (2) is less than 3. So, take the first two numbers of the dividend (20) and determine how many times 3 will go into 20.

$$3 \times 6 = 18 \text{ and } 3 \times 7 = 21$$

Thus 3 will go into 20 six times. Place the 6 above the division bracket over the 0 from the 20.

$$\begin{array}{r} 6 \\ 3\overline{)2067} \end{array}$$

- Multiply the 6 by 3 and place the result below the 20 of the dividend.

$$\begin{array}{r} 6 \\ 3\overline{)2067} \\ 18 \end{array}$$

- Draw a line under the 18 and subtract it from 20 (20-18=2). Bring down the 6 from the 2067 and place it to the right of the 2.

$$\begin{array}{r} 6 \\ 3\overline{)2067} \\ \underline{18} \\ 26 \end{array}$$

- Determine how many times 3 will go into 26 and place the answer above the division bracket and to the right of the six.

$$\begin{array}{r} 68 \\ 3\overline{)2067} \\ \underline{18} \\ 26 \end{array}$$

- Multiply the 8 by the 3 and place the result below the 26. Subtract the 24 from the 26. Bring down the 7 and place it next to the 2.

$$\begin{array}{r} 68 \\ 3\overline{)2067} \\ \underline{18} \\ 26 \\ \underline{24} \\ 27 \end{array}$$

- Multiply the 9 by the 3 and place the result below the 27. Subtract the 27 from the 27.

$$\begin{array}{r} 689 \\ 3\overline{)2067} \\ \underline{18} \\ 26 \\ \underline{24} \\ 27 \\ \underline{27} \\ 0 \end{array}$$

- There are no more numbers in the dividend to bring down next to the 0. Therefore 3 divided evenly into 2067.

$$\begin{array}{r} 689 \\ 3\overline{)2067} \end{array}$$

Real World - Whole Numbers

Below are some helpful hints to use when solving word problems.

UNDERSTAND THE PROBLEM
• *Identify what to solve for in the problem.*
• *Weed out any unnecessary information given.*
DEVISE A PLAN
• *What steps need to be taken to solve the problem?*
• *Identify the skills that can be applied to solving the problem.*
CARRY OUT THE PLAN
• *Perform the necessary calculations to solve the problem.*
• *Make sure to answer the question being asked.*
REVIEW YOUR WORK
• *Does the answer seem reasonable?*
• *Check for careless mistakes.*
(Adapted from Polya's Four-Step Process)

EXAMPLES

1. **Directions: Type the correct answer in the box. Use numerals instead of words.**

   At True Sound Electronics, Eric can assemble 55 of their newest model of mp3 players in one hour. A large department store just placed an order for 8,382 of the mp3 players in preparation for the holiday season. Eric has been asked to fill this order by himself because everyone else is busy working on assembling video recorders.

   Without any distractions, it is going to take Eric [   ] hours to fill the order.

   Submit

3.

$$315 \div 15 = ?$$

A 22

B 21

C 300

D 23

2.   709
     × 371

[   ]   **Submit**

Compute Decimal Solutions

Adding Decimals

1. Place the numbers so that the decimal points are aligned vertically.

2. Add each column, starting on the right and working left.

3. If the sum of a column is greater than or equal to ten, carry the one to the next column on the left.

Find the sum of the following numbers.

$$34.87 + 16.845 = ?$$

Solution:

Write the numbers so that their decimal points are aligned vertically. Then, add.

$$\begin{array}{r} \overset{1\ \ 1\ \ 1}{34.870} \\ +\ 16.845 \\ \hline 51.715 \end{array}$$

Subtracting Decimals

1. Place the numbers so that the decimal points are aligned vertically.(Add zeros if needed.)

2. Subtract each column, starting on the right and working left.

3. If the number being subtracted is larger than the number it is being subtracted from, subtract one from the number in the next left column, and then add ten to the number. (This is called regrouping, or borrowing.)

Multiplying Decimals

1. Multiply the numbers, ignoring the decimal points.

2. Count the total number of places after the decimals in the numbers being multiplied.

3. The answer will have the same number of places after the decimal as the total number of places in the numbers being multiplied.

Example 1:

Find the product of the following numbers.

$$0.245 \times 3.8 = ?$$

Solution:

Multiply the numbers, ignoring the decimal points. Since 0.245 has 3 decimal places and 3.8 has 1 decimal place, the answer will have a total of 4 decimal places.

```
      0.245
  x     3.8
       1960
       7350
     0.9310
```

Dividing Decimals

1. Move the decimal point in the divisor to the right to make it a whole number.

2. Move the decimal point in the dividend to the right the same number of places the decimal was moved for the divisor.

3. Divide the numbers ignoring the decimal point.

4. Bring the decimal point straight up from the dividend into the answer.

Find the quotient of the following numbers.

$$93.75 \div 7.5 = ?$$

Solution:

To make the divisor, 7.5, a whole number, move the decimal point 1 place to the right. The divisor is now 75. Move the decimal point in the dividend, 93.75, 1 place to the right as well. The dividend is now 937.5.

Now, divide and bring the decimal point straight up in the answer.

```
              12.5
    7.5.)93.7.5
         75
         187
         150
          375
          375
            0
```

VIDEOS:

Adding Decimals https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/adding-decimals-pre-alg/v/adding-decimals?v=w616LEmrHVE

Subtracting Decimals: https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/adding-decimals-pre-alg/v/subtracting-decimals?v=Eq4mVCd-yyo

Multiplying Decimals: https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-arithmetic-operations/cc-6th-multiplying-decimals/v/multiplying-decimals?v=JEHejQphIYc

Dividing Decimals: https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-arithmetic-operations/cc-6th-div-decimals/v/dividing-decimal?v=S0uuK7SQcA8

EXAMPLES:

1.
Simplify the following expression.

$19.7 + 4.2$

A   23.9

B   61.7

C   6.17

D   20.12

2. Divide.

$0.07 \div 0.07$

A   0.07

B   0.007

C   100

D   1

Number Theory

Factors
the numbers multiplied together to get another number

Greatest Common Factor (GCF)
the largest number that is a common factor of two or more numbers

Prime Factorization
the expression of a positive integer as a product of prime numbers

Example

What is the greatest common factor (GCF) of 20 and 30?

Solution

Find the prime factorization of each number.

$$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$$
$$30 = 2 \times 3 \times 5 = 2^1 \times 3^1 \times 5^1$$

To find the greatest common factor, multiply together the highest power of each prime that both numbers share.
The highest power of 2 that both 20 and 30 share is 1.
The highest power of 5 that both 20 and 30 share is 1.
So, the GCF is $2^1 \times 5^1 = \mathbf{10}$.

Least Common Multiple (LCM) is the smallest number that is a multiple of two or more numbers.

Example:

What is the least common multiple (LCM) of 12 and 10?

Solution:

List the multiples of 12 and 10 until there is one in common.

12 = 12, 24, 36, 48, 60
10 = 10, 20, 30, 40, 50, 60

So, the LCM is **60**.

Distributive Property
$a(b + c) = (a \times b) + (a \times c)$
The **common factor** in the example above is **a**.

VIDEO: https://www.khanacademy.org/math/pre-algebra/factors-multiples/greatest_common_divisor/v/greatest-common-divisor?v=jFd-6EPfnec

1. What is the greatest common factor (GCF) of 66 and 99?

   A   22

   B   11

   C   33

   D   198

. What is the least common multiple (LCM) of 8 and 10?

   A   80

   B   2

   C   40

   D   160

2. **Directions: Type the correct answer in each box. Use numerals instead of words. If necessary, use decimals instead of fractions.**

   Harry is making gift bags for a holiday party using two types of candies. He has a total of 84 chocolate candies and 56 peppermint candies. Each gift bag he makes will be the same.

   If Harry uses all of his candy, he can make at most ☐ gift bags. Each gift bag will have ☐ chocolate candies and ☐ peppermint candies.

Number Sentences

Translating verbal descriptions into mathematical equations can be helpful for problem solving.

Example:

Jeremy went to the Die-Cast Car collectors convention and bought 7 cars for 56 dollars. If each car costs the same amount, find the number sentence that can be used to find the cost of each car.

Solution:

Since the cost of each car is unknown, let c stand for the cost of each car.

Multiplying the number of cars bought, 7, by the number of dollars each car cost, c, will tell the total amount Jeremy spent on the cars in all.

$$7 \times c = ?$$

The question says that he bought all of the cars for 56 dollars. Since 56 is the product of 7 and c, it goes on the right side of the equal sign.

So, the number sentence that will help solve the problem is shown below.

$$7 \times c = 56$$

> *To solve a one-step equation, get the variable by itself on one side of the equation.*
> *To isolate the variable, use an opposite operation.*
> *To keep the equation balanced, perform the same operation on both sides of the equation.*

Example:

Solve the following equation for *x*.

$$x + 16.8 = 37.5$$

Solution:

Use an opposite operation to isolate *x*.

$$x + 16.8 = 37.5$$
$$x + 16.8 - 16.8 = 37.5 - 16.8$$
$$x = 20.7$$

Substitute this value back into the original equation to check that the answer is correct.

$$20.7 + 16.8 = 37.5$$

> *A **number line** can be used to graph an **inequality** in one variable.*

|	
Read < as "less than".	Read ≤ as "less than or equal to".
Read > as "greater than".	Read ≥ as "greater than or equal to".

Example 1:

Graph the inequality *x* > 1 on a number line.

Solution:

The inequality is greater than, so an open circle is used on the number line. The open circle will be on one, and a line should extend to the right as the numbers increase on the number line.
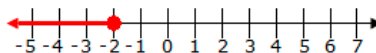
The graph of the inequality is shown below.
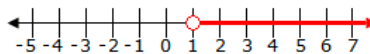
Example 2:

Graph the inequality *x* ≤ -2.

Solution:

The inequality is less than or equal to, so a closed circle is used on the number line. The closed circle will be on negative two, and a line should extend to the left as the numbers decrease on the number line.

The graph of the inequality is shown below.

VIDEOS:

https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-expressions-and-variables/cc-6th-beginner-equations/v/solving-one-step-equations?v=VidnbCEOGdg

https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-expressions-and-variables/cc-6th-beginner-equations/v/simple-equations?v=9Ek61w1LxSc

https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-expressions-and-variables/cc-6th-inequalities/v/inequalities-on-a-number-line?v=dTwZ5N126gw

EXAMPLES:

1. Milo's Ice Cream Factory sold more than 45 waffle cones yesterday. Which of the following inequalities represents the number of waffle cones Milo's Ice Cream Factory sold yesterday?

   A $x < 45$

   B $x = 45$

   C $x < 46$

   D $x > 45$

. Wanda has two times more shirts than her brother, Will.

The equation $a = 2b$, where a represents the number of shirts Wanda has, and b represents the number of shirts Will has, shows this relationship.

If Wanda has 16 shirts, how many shirts does Will have?

   A 14

   B 8

   C 15

   D 7