6.4 Study Island Guide

a. Units of Measurement

Convert Units

In measurement problems, it is sometimes important to convert from one unit to another within a system.

UNITS OF LENGTH

Customary
12 inches = 1 foot
3 feet = 1 yard
5,280 feet = 1 mile
1,760 yards = 1 mile

Metric
1,000 millimeters = 1 meter
100 centimeters = 1 meter
10 millimeters = 1 centimeter
1 kilometer = 1,000 meters

Example:

Convert 5 yards to feet.

Solution:

Since 3 feet equals 1 yard, use that ratio to find how many feet is equal to 5 yards.

$$5 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} = 15 \text{ feet}$$

So, **15 feet** is equal to 5 yards.

In measurement problems, it is sometimes important to convert from one unit to another within a system.

UNITS OF CAPACITY

Customary
8 fluid ounces = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

Metric
1,000 milliliters = 1 liter

Example:

Convert 16 cups to pints.

Solution:

Since 1 pint equals 2 cups, use that ratio to find how many pints is equal to 16 cups.

$$16 \text{ cups} \div \frac{2 \text{ pints}}{1 \text{ cup}} = 8 \text{ pints}$$

So, **8 pints** is equal to 16 cups.

Use the customary and metric conversion charts below to convert weight measurements from one unit to another within the same measurement system.

Customary Units

1 ton	=	2,000 pounds
1 pound	=	16 ounces

Metric Units

1 gram	=	1,000 milligrams
1 kilogram	=	1,000 grams

Example 1:

Convert 1.375 kilograms to grams.

Solution:

Use the following conversion factor to convert 1.375 kilograms to grams.

1 kg = 1,000 g

Multiply to convert from kilograms to grams.

$$1.375 \text{ kg} \times \frac{1{,}000 \text{ g}}{1 \text{ kg}} = 1{,}375 \text{ g}$$

Therefore, 1.375 kilograms is equal to **1,375 grams**.

Example 2:

Convert 7,500 pounds to tons.

Solution:

Use the following conversion factor to convert 7,500 pounds to tons.

1 ton = 2,000 lb

Multiply to convert from pounds to tons.

$$7{,}500 \text{ lb} \times \frac{1 \text{ ton}}{2{,}000 \text{ lb}} = 3.75 \text{ tons}$$

Therefore, 7,500 pounds is equal to **3.75 tons**.

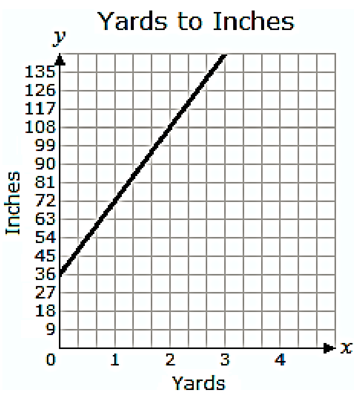
Equivalent ratios relating quantities with whole-number measurements can be displayed in a table or in a graph on the coordinate plane.

Example:

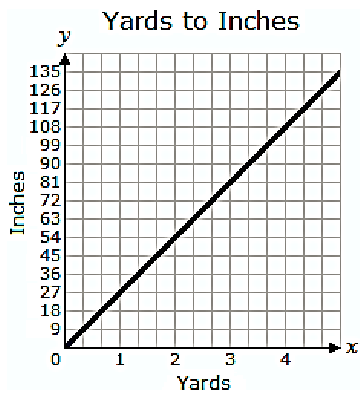
Use the table below to determine the relationship between yards and inches.

Yards	Inches
1	36
2	?
3	?
4	?

Which of the following graphs matches the relationship shown in the table?



W.



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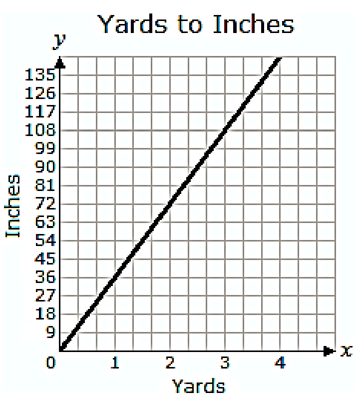
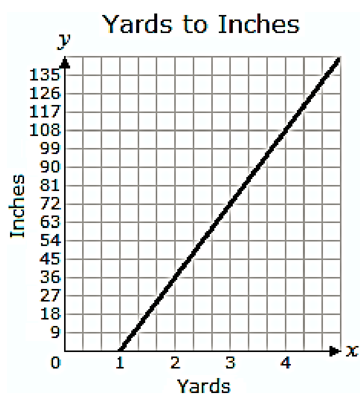


Y.



Z.

Solution:

The table below shows the relationship between the number of yards and the number of inches. Since there are 36 inches in 1 yard, multiply the number of yards by 36.

Yards	Yards × 36	Inches
1	1 × 36 = 36	36
2	2 × 36 = 72	72
3	3 × 36 = 108	108
4	4 × 36 = 144	144

The graph which matches the table will have the coordinates (1,36), (2,72), (3,108), and (4,144).

Therefore, the correct graph is **Y**.

When comparing measurements given in a table, write the ratios as units fractions. The fraction representing the ratio with the smallest denominator will be the largest ratio.

Example:

Yards	11	13
Feet	33	39

Which of the following tables represents a ratio which is greater than the ratio in the table above?

A.

Quarts	4	6
Pints	8	12

B.

Weeks	2	5
Days	14	35

Solution:

Determine the ratio of each table.

$$\frac{11\ \text{yd}}{33\ \text{ft}} = \frac{1}{3}\ \frac{\text{yd}}{\text{ft}}$$
$$\frac{4\ \text{qt}}{8\ \text{pt}} = \frac{1}{2}\ \frac{\text{qt}}{\text{pt}}$$
$$\frac{2\ \text{wk}}{14\ \text{days}} = \frac{1}{7}\ \frac{\text{wk}}{\text{days}}$$

Next, compare the ratios. Since all the ratios are unit fractions, the smallest denominator will be the largest fraction.

$$\frac{1}{7} < \frac{1}{3} < \frac{1}{2}$$

Therefore, the table with a ratio greater than the given table is table **A**.

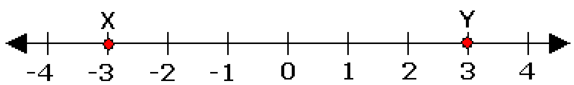
Quarts	4	6
Pints	8	12

b. Absolute Value

Absolute Value

*The **absolute value** of a number is the distance from the number to zero on a number line.*

Example:



The absolute value at point of **Y**, notated as |**Y**|, is 3 because point **Y** is 3 units from zero.

$$|3| = 3$$

The absolute value at point **X**, notated as |**X**| is also 3 because point **X** is 3 units from zero.

$$|-3| = 3$$

Therefore, |**Y**| = |**X**| because they are the same distance from zero.

*The **absolute value** of a number is its distance from zero on a number line.*

Absolute value is often used in real world problems to describe a situation.

Example:

The temperature at midnight was less than -8°F.

Which of the following best describes the temperature?

A. The temperature was warmer than 8 degrees above 0.
B. The temperature was warmer than 8 degrees below 0.
C. The temperature was colder than 8 degrees above 0.
D. The temperature was colder than 8 degrees below 0.

Solution:

Having positive and negative values, a thermometer is similar to a number line.

Temperatures with a negative value represent a temperature below zero.
Temperatures with a positive value represent a temperature above zero.
Since the temperature is negative, the temperature is below zero.
To determine how far below zero the temperature was, find the absolute value of -8°F.

$$|-8^{\circ}\mathrm{F}| = 8^{\circ}\mathrm{F} \text{ below } 0$$

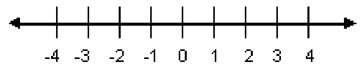
A temperature of -8°F is equal to 8°F below zero.
If a temperature is less than -8°F, then the temperature is colder than 8°F below zero.
Therefore, **the temperature was colder than 8 degrees below 0**.

c. Negative and Positive Numbers

On a number line, 0 is the only number that is neither positive nor negative.
*All numbers to the right of 0 are **positive** numbers.*
*All numbers to the left of 0 are **negative** numbers.*
*Negative numbers have a **negative sign** in front of them:*
-4, -6, -12, etc.

Number Line

Below is an example of what a **number line** looks like.



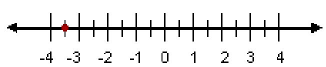
The farther to the *right* a number is on the number line, the *greater* it is.

The farther to the *left* a number is on the number line, the *smaller* it is.

A positive number is always greater than a negative number.

Question 1:

What number is indicated by the dot on the number line?



Solution:

The dot is between -3 and -4. The tick marks divide the space between numbers into halves.

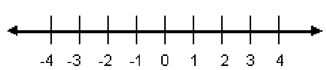
The number halfway between -3 and -4 is **-3.5**, or $-3\frac{1}{2}$.

Question 2:

Start with zero and count backwards two. What is that number?

Solution:

Starting with zero and counting backwards, the first number is -1, the next number is **-2**.



Question 3:

How is negative seven-tenths written numerically?

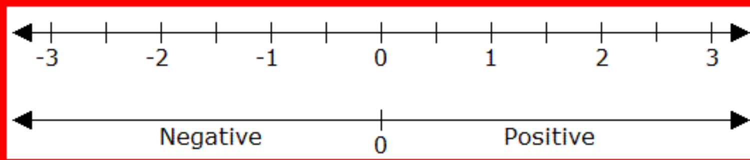
Solution:

The negative of a value is written the same as the positive of the value, but with a negative sign in front of it.

Therefore, negative seven-tenths written numerically is **-0.7**, or $-\frac{7}{10}$.

Comparing Rational Quantities

When comparing positive and negative rational quantities, remember that the farther the number is to the left of zero on a number line, the smaller the number is. The farther the number is to the right of zero, the larger the number is.



Example 1:

The temperature in Anchorage is -12° F, and the temperature in Dallas is 65° F. Write an inequality to compare the temperatures of the two cities.

Solution:

Negative values are always less than positive values.

So, -12° F is less than 65° F.

The symbol for "less than" is "<".

An inequality that correctly compares the temperatures of the two cities is shown below.

Anchorage's temperature < Dallas's temperature

Example 2:

Kim's account balance is -$34.55, and Nick's account balance is -$36.75. Write an inequality to compare the account balances of Kim and Nick.

Solution:

Negative values with smaller absolute values are farther to the right on a number line than negative values with larger absolute values.

This means that negative values with smaller absolute values are greater than negative values with larger absolute values.

So, -$34.55 is greater than -$36.75.

The symbol for "greater than" is ">".

An inequality that correctly compares the account balances is shown below.

Kim's account balance > Nick's account balance

VIDEO: Negative Numbers

https://www.khanacademy.org/math/arithmetic/absolute-value/add-sub-negatives/v/negative-numbers-introduction?v=Hlal9ME2Aig

d. Area

Area *is the amount of space taken up by a flat (2-dimensional) shape.*

FORMULAS

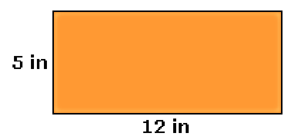
$$\text{Area of a Rectangle} = \text{length} \times \text{width}$$

$$\text{Area of a Square} = \text{side}^2$$

$$\text{Area of a Triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

Example 1:

Find the area of the rectangle below.



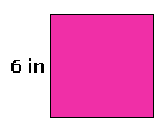
Solution:

To find the area of the rectangle, use the formula below.

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= 5 \text{ in} \times 12 \text{ in} \\ &= 60 \text{ in}^2 \end{aligned}$$

Example 2:

Find the area of the square below.



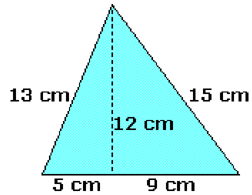
Solution:

To find the area of the square, use the formula below.

$$\begin{aligned} \text{Area} &= \text{side}^2 \\ &= (6 \text{ in})^2 \\ &= 36 \text{ in}^2 \end{aligned}$$

Example 3:

Find the area of the triangle below.



Solution:

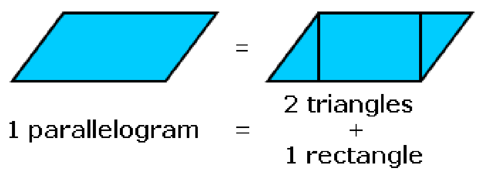
To find the area of the triangle, use the formula below.

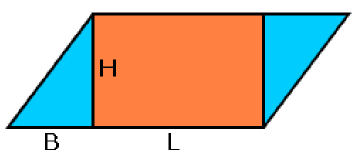
$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times \text{base} \times \text{height} \\ &= \tfrac{1}{2} \times (5\text{ cm} + 9\text{ cm}) \times 12\text{ cm} \\ &= \tfrac{1}{2} \times 14\text{ cm} \times 12\text{ cm} \\ &= 84\text{ cm}^2\end{aligned}$$

*The area of a **parallelogram** can be found in two ways.*

1. Split the parallelogram into two triangles and a rectangle, and add up the area of each section.
or
2. Use the formula for the area of a parallelogram.

Split the parallelogram into two triangles and a rectangle.





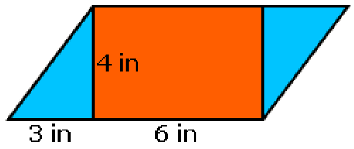
$\text{Area}_{\text{triangle}}$ = (base × height) ÷ 2

$\text{Area}_{\text{rectangle}}$ = length × width

$\text{Area}_{\text{parallelogram}}$ = $(2 \times \text{Area}_{\text{triangle}}) + \text{Area}_{\text{rectangle}}$

Example 1:

Find the area of the parallelogram below.



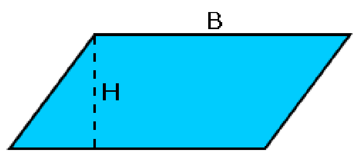
Solution

$$\text{Area}_{\text{triangle}} = (3 \text{ in} \times 4 \text{ in}) \div 2$$
$$= 12 \text{ in}^2 \div 2$$
$$= 6 \text{ in}^2$$

$$\text{Area}_{\text{rectangle}} = 6 \text{ in} \times 4 \text{ in}$$
$$= 24 \text{ in}^2$$

$$\text{Area}_{\text{parallelogram}} = (2 \times 6 \text{ in}^2) + 24 \text{ in}^2$$
$$= 12 \text{ in}^2 + 24 \text{ in}^2$$
$$= 36 \text{ in}^2$$

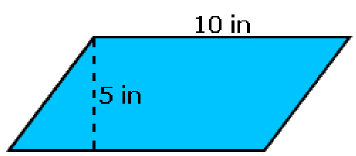
The next example uses a formula to find the area of a parallelogram.



Area = $B \times H$

Example 2:

Find the area of the parallelogram below.



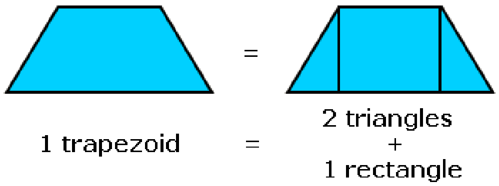
Solution

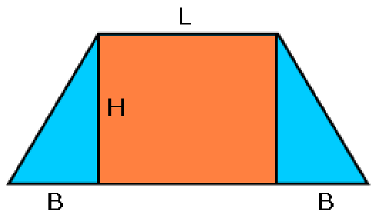
$$\text{Area} = B \times H$$
$$= 10 \text{ in} \times 5 \text{ in}$$
$$= 50 \text{ in}^2$$

Area of a Trapezoid

*The **area of a trapezoid** can be found in two ways.*

One way to find the area of a trapezoid is to split the trapezoid into two triangles and a rectangle, and then add up the area of each section.





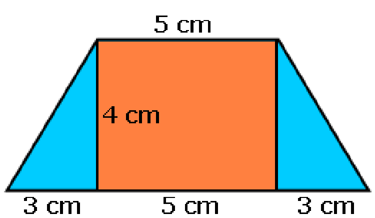
$\text{Area}_{\text{triangle}} = (B \times H) \div 2$

$\text{Area}_{\text{rectangle}} = L \times W$

$\text{Area}_{\text{trapezoid}} = (2 \times \text{Area}_{\text{triangle}}) + \text{Area}_{\text{rectangle}}$

Example 1:

Find the area of the trapezoid below.



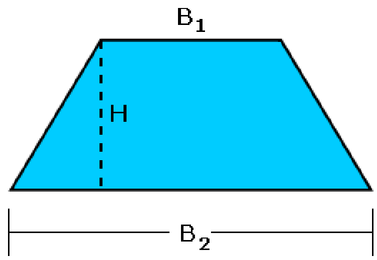
Solution:

$\text{Area}_{\text{triangle}} = (3 \text{ cm} \times 4 \text{ cm}) \div 2 = 12 \div 2 = 6 \text{ cm}^2$

$\text{Area}_{\text{rectangle}} = 5 \text{ cm} \times 4 \text{ cm} = 20 \text{ cm}^2$

$\text{Area}_{\text{trapezoid}} = (2 \times 6 \text{ cm}^2) + 20 \text{ cm}^2 = 12 \text{ cm}^2 + 20 \text{ cm}^2 = \mathbf{32 \text{ cm}^2}$

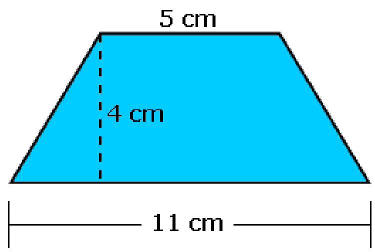
Another way to find the area of a trapezoid is to use the formula below.



$$\text{Area} = \frac{1}{2} \times (B_1 + B_2) \times H$$

Example 2:

Find the area of the trapezoid below.



Solution:

$$\text{Area} = \frac{1}{2} \times (B_1 + B_2) \times H$$

$$\text{Area} = \frac{1}{2} \times (5\text{ cm} + 11\text{ cm}) \times 4\text{ cm}$$

$$\text{Area} = \frac{1}{2} \times (16\text{ cm}) \times 4\text{ cm}$$

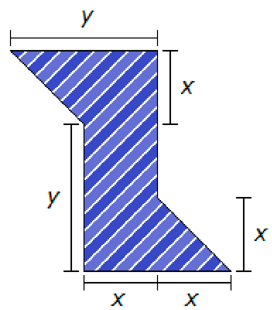
Area = $\mathbf{32\ cm^2}$

Composite Figures - Area

To find the ***area*** *of a composite figure,*
break the figure up into smaller shapes.

Example 1:

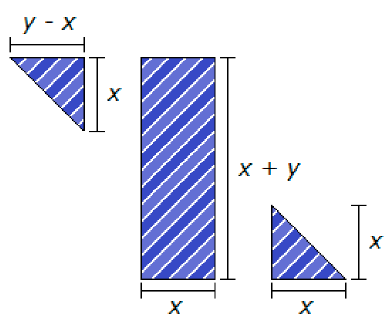
Zane designed the T-shirt logo shown below.



If x = 4 inches and y = 8 inches, what is the area of the logo?

Solution:

Since this is an unusual shape, break it up into a rectangle and two triangles.



Now, find the area of the rectangle and the area of the two triangles.

$$\begin{aligned} A_{\text{rectangle}} &= (\text{length})(\text{width}) \\ &= (x + y)(x) \\ &= (4 \text{ in} + 8 \text{ in})(4 \text{ in}) \\ &= (12 \text{ in})(4 \text{ in}) \\ &= 48 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{top triangle}} &= \tfrac{1}{2}(\text{base})(\text{height}) \\ &= \tfrac{1}{2}(y - x)(x) \\ &= \tfrac{1}{2}(8 \text{ in} - 4 \text{ in})(4 \text{ in}) \\ &= \tfrac{1}{2}(4 \text{ in})(4 \text{ in}) \\ &= 8 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{bottom triangle}} &= \tfrac{1}{2}(\text{base})(\text{height}) \\ &= \tfrac{1}{2}(x)(x) \\ &= \tfrac{1}{2}(4 \text{ in})(4 \text{ in}) \\ &= 8 \text{ in}^2 \end{aligned}$$

Finally, add the areas of the rectangle and the two triangles.

$$\begin{aligned} \text{Area} &= 48 \text{ in}^2 + 8 \text{ in}^2 + 8 \text{ in}^2 \\ &= 64 \text{ in}^2 \end{aligned}$$

e. Ratios and Ratio Language

Ratios and Ratio Language

A ratio represents a comparison between two values.

A ratio of "a to b" can be written in the following ways.
a to b
a:b
$\frac{a}{b}$

Example 1:

Melissa has a candy jar filled with some chocolate candies and some mint drops.
There are 15 chocolates and 20 mints.
What is the ratio of chocolate candies to mint drops?

Solution:

Rewrite the information in ratio form.

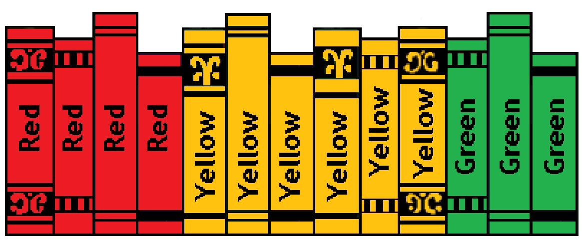
15 to 20, or **15:20**, or $\frac{15}{20}$

These ratios could also be written in simplified form, by dividing all values by 5.

3 to 4, or 3:4, or $\frac{3}{4}$

Example 2:

What is the ratio of green books to red books?



Solution:

Since there are 3 green books and 4 red books, the ratio of green books to red books is **3 to 4**.

f. Division of Fractions

Division of Fractions

Dividing Fractions

1. Rewrite the division as multiplication by the reciprocal of the divisor.

2. Multiply the numerators and the denominators.

3. Reduce, if possible.

Example 1:

Divide the following.

$$\frac{5}{6} \div \frac{2}{3}$$

Solution:

To divide, flip the second fraction, and then multiply.

$$\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{15}{12}$$

Simplify when possible.

$$\frac{15}{12} = \frac{5}{4} = 1\frac{1}{4}$$

Example 2:

Divide the following.

$$2\frac{3}{4} \div 1\frac{1}{2}$$

Solution:

When dividing mixed numbers, first, convert the mixed numbers to improper fractions.

$$2\frac{3}{4} \div 1\frac{1}{2} = \frac{11}{4} \div \frac{3}{2}$$

Next, to divide, flip the second fraction, and then multiply.

$$\frac{11}{4} \div \frac{3}{2} = \frac{11}{4} \times \frac{2}{3} = \frac{22}{12}$$

Simplify when possible.

$$\frac{22}{12} = \frac{11}{6} = 1\frac{5}{6}$$

VIDEO: Dividing Fractions: https://www.khanacademy.org/math/arithmetic/fractions/div-fractions-fractions/v/dividing-fractions-example?v=tnkPY4UqJ44

Dividing Whole # and Fractions: https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-arithmetic-operations/cc-6th-div-whole-fractions/v/dividing-fractions-word-problem?v=PQsgXNggV7Q

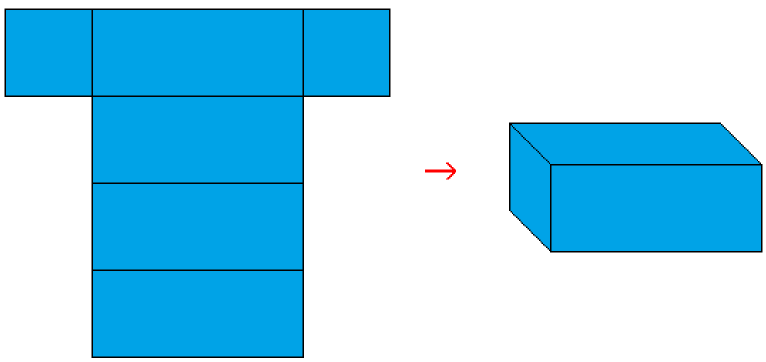
Surface Area and Volume

Surface area *is the measure of the number of square units needed to cover the outside of a figure. In other words, it is the area of the surface of an object.*

To calculate the surface area of an object, sum the areas of the faces of the object.

Example 1:

The rectangular prism below was constructed from the pattern given. It is 12 inches in length, 4 inches wide, and 4 inches tall. Find the surface area of the rectangular prism.



Solution:

The figure is a rectangular solid with 4 long sides and 2 short sides.

The long sides have a width of 4 inches and a length of 12 inches. The area of each long side is shown below.

$$\begin{aligned} \text{Area}_{\text{long side}} &= \text{length} \times \text{width} \\ &= 12 \text{ in} \times 4 \text{ in} \\ &= 48 \text{ in}^2 \end{aligned}$$

Since there are 4 long sides, multiply 48 in^2 by 4.

$$48 \text{ in}^2 \times 4 = 192 \text{ in}^2$$

Next, find the area of each short side. The short sides have a width of 4 inches and a height of 4 inches. The area of each short side is shown below.

$$\begin{aligned} \text{Area}_{\text{short side}} &= \text{height} \times \text{width} \\ &= 4 \text{ in} \times 4 \text{ in} \\ &= 16 \text{ in}^2 \end{aligned}$$

Since there are 2 short sides, multiply 16 in^2 by 2.

$$16 \text{ in}^2 \times 2 = 32 \text{ in}^2$$

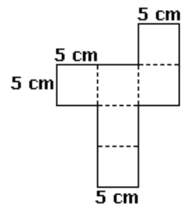
Now, add the total area of the long sides to the total area of the short sides.

$$192 \text{ in}^2 + 32 \text{ in}^2 = 224 \text{ in}^2$$

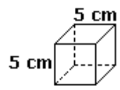
Therefore, the surface area of the rectangular prism shown by the pattern above is **224 in^2**.

Example 2:

Find the surface area of the geometric shape formed by the pattern below.



Solution:



The pattern shown forms a cube.

Find the area of one side of the cube by using the formula below.

$$\text{Area} = \text{length} \times \text{width}$$
$$= 5\text{ cm} \times 5\text{ cm}$$
$$= 25\text{ cm}^2$$

So, the area of one face of the cube is 25 cm^2.

Since a cube has six square faces, multiply the area of one square face, 25 cm^2, by six to find the surface area of the cube.

$$25\text{ cm}^2 \times 6 = 150\text{ cm}^2$$

Therefore, the surface area of the geometric shape shown by the pattern above is **150 cm^2**.

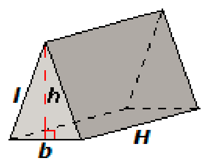
Surface Area

*The **surface area** of a solid is the measure of the number of square units needed to cover the outside of the solid. In other words, it is the area of the surface of a solid.*

To find the surface area of a triangular prism, find the area of each face and then add them together.

Example:

Find the surface area of the isosceles triangular prism below. The base (b) of the triangle measures 6 inches, the height (h) measures 4 inches, and the side length (l) measures 5 inches. The height (H) of the triangular prism is 8 inches.



Solution:

In the triangular prism, there are 2 congruent triangular faces and 3 rectangular faces.

Since the two upper rectangular faces have the same length and width, they will have the same area.

First, find the area of one of the bases.

$$\begin{aligned} \text{Area of a Triangle Base} &= \tfrac{1}{2} \times \text{base} \times \text{height} \\ &= \tfrac{1}{2} \times 6 \text{ inches} \times 4 \text{ inches} \\ &= 12 \text{ inches}^2 \end{aligned}$$

Second, find the area of one of the upper rectangular faces.

$$\begin{aligned} \text{Area of an Upper Rectangular Face} &= \text{length} \times \text{Height} \\ &= 5 \text{ inches} \times 8 \text{ inches} \\ &= 40 \text{ inches}^2 \end{aligned}$$

Third, find the area of the lower rectangular face.

$$\begin{aligned} \text{Area of the Lower Rectangular Face} &= \text{base} \times \text{Height} \\ &= 6 \text{ inches} \times 8 \text{ inches} \\ &= 48 \text{ inches}^2 \end{aligned}$$

Last, multiply the area of one of the triangular faces by 2, multiply the area of one of the upper rectangular faces by 2, and then add those numbers and the area of the lower rectangular face together.

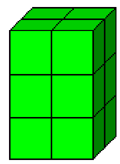
$$\begin{aligned} \text{Surface Area} &= (12 \text{ inches}^2 \times 2) + (40 \text{ inches}^2 \times 2) + 48 \text{ inches}^2 \\ &= 24 \text{ inches}^2 + 80 \text{ inches}^2 + 48 \text{ inches}^2 \\ &= 152 \text{ inches}^2 \end{aligned}$$

Surface Area and Volume

When finding the volume of 3-D shapes made with cubes, find out how many cubes are on each layer, then add them all up.

Example:

The prism below is made of cubes which measure $\frac{1}{3}$ of a centimeter on each side. What is the volume?



Note: Figure not drawn to scale.

Solution:

First, find the volume of each cube.

$$\frac{1}{3}\text{ cm} \times \frac{1}{3}\text{ cm} \times \frac{1}{3}\text{ cm} = \frac{1}{27}\text{ cubic cm}$$

Now, count how many cubes are in the shape. Since there are 3 layers of cubes and 4 cubes in each layer, there are 12 cubes in the shape. So, multiply the volume of one cube by the number of cubes in the shape to find the volume.

$$\frac{1}{27}\text{ cubic cm} \times 12 = \frac{12}{27}\text{ cubic cm}$$

Finally, reduce the fraction.

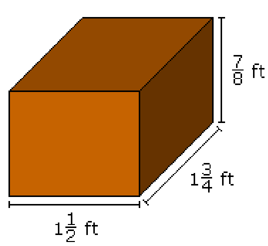
$$\frac{12}{27}\text{ cubic cm} = \frac{4}{9}\text{ cubic cm}$$

Volume can also be found using the following formula where the length is l, the width is w, and the height is h.

$$V = l \times w \times h$$

Example:

What is the volume of the box below?



Note: Figure not drawn to scale.

Solution:

The formula for the volume of a rectangular prism is shown below.

$$V = \text{length} \times \text{width} \times \text{height}$$

To find the volume of the box, substitute the values given in the question into the formula.

$$\begin{aligned} V &= \left(1\tfrac{1}{2}\text{ ft}\right) \times \left(1\tfrac{3}{4}\text{ ft}\right) \times \left(\tfrac{7}{8}\text{ ft}\right) \\ &= \left(\tfrac{3}{2}\text{ ft}\right) \times \left(\tfrac{7}{4}\text{ ft}\right) \times \left(\tfrac{7}{8}\text{ ft}\right) \\ &= \tfrac{147}{64}\text{ cubic ft} \\ &= 2\tfrac{19}{64}\text{ cubic ft} \end{aligned}$$